8.2 : 2 prop

- compare 2 distinct pop.
- a distinct sample
- independent

- compare $\hat{p}_1$ and $\hat{p}_2$
- $\hat{p}_1 = \hat{p}_2$ $\hat{p}_1 - \hat{p}_2 = 0$
- the diff. btw. $\hat{p}_1$ + $\hat{p}_2$

Generic

Stat $\pm$ (crit. value)(Std dev. of stat.)

2 prop Z-Int.

specific

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \underbrace{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}_{\text{Std. error}}$$

We are ____% confident that the difference btw. prop #1 and #2 is btw. _____ and _____.

Test

1) Assump
2) Hyp.
3) test stat.
4) p-value
5) conclusion

Pop 1		Pop 2
p_1	unknown	p_2
$\hat{p}_1$		$\hat{p}_2$
n_1		n_2

given $\left\{ \hat{p}_1, n_1 \right.$

Hyp

Comparing 2 pop. prop.

$H_0: p_1 = p_2$ $\quad$ $p_1 - p_2 = 0$

OR

$H_a: p_1 \overset{>}{\underset{\neq}{<}} p_2$ $\quad$ $p_1 - p_2 \overset{>}{\underset{\neq}{<}} 0$

Generic

$$\frac{\text{stat.} - \text{param.}}{\left(\begin{array}{c}\text{std. dev.}\\ \text{of stat.}\end{array}\right)}$$

not in conf. int. → $H_0: p_1 = p_2$

$\hat{p}_1 \quad \hat{p}_2$

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2}$$

$$\neq \hat{p}_1 + \hat{p}_2$$

0.45 0.65

$n = 100$ $n = 100$

Specific

$$z = \frac{(\hat{p}_1 - \hat{p}_2)}{\underbrace{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}_{\text{Std. error}}}$$

*pooled

2 prop Z test

P-value:

$P(z \geq \underline{\text{test stat}}) =$

Concl.

- same
- We have suff. evid. that the prop of $\underline{\#1}$ is $\underline{> < \neq}$ the prop. of $\underline{\#2}$.

Assump

1) 2 indep SRS

2) $n_1 \hat{p}_1$
$n_1(1-\hat{p}_1)$
$n_2 \hat{p}_2$
$n_2(1-\hat{p}_2)$
≥ 10

3) $Pop_1 \geq 10 \cdot n_1$
$Pop_2 \geq 10 \cdot n_2$

Example

$\hat{p}_M = 0.227$

$\hat{p}_W = 0.1698$

State	Check
1) 2 indep. SRS	1) assumed
2) $n_1\hat{p}_1$, $n_1(1-\hat{p}_1)$, $n_2(1-\hat{p}_2)$, $n_2\hat{p}_2$ ≥ 10	2) 1630, 5550, 1684, 8232 ≥ 10 ✓
3) $pop_1 \geq 10 \cdot n_1$, $pop_2 \geq 10 \cdot n_2$	3) $pop_M \geq 71800$ ✓, $pop_W \geq 99160$ ✓

$H_0: p_M = p_W$

$H_a: p_M \neq p_W$

$$z = \frac{\hat{p}_M - \hat{p}_W}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_W} + \frac{1}{n_M}\right)}} = 9.337$$

$2 \cdot P(z > 9.337) = 1.0112 \times 10^{-20}$

- We reject Ho b/c p-value $< \alpha = 0.05$
- We have suff. evid. that the prop. of male binge drinkers is not equal to the prop. of female binge drinkers.

Assump - checked above

$$(\hat{p}_M - \hat{p}_W) \pm z^* \sqrt{\frac{\hat{p}_M(1-\hat{p}_M)}{n_M} + \frac{\hat{p}_W(1-\hat{p}_W)}{n_W}}$$

$$= (0.04631, 0.06808)$$

We are 92% conf. that the difference between the percent of male & female binge drinkers is btw. 4.631% and 6.808%